

Tear it up!

investigating geometric
series & limits



Teacher
Notes

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<http://smartboardsmarty.wikispaces.com>

What's the answer?

Take a minute and think about
how you might solve this problem!

$$\frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \frac{1}{16} + \dots =$$



You can find the answer if you
have a piece of paper.

You'll need one piece of paper and a writing
utensil. When you are ready, go to the next page
and begin.

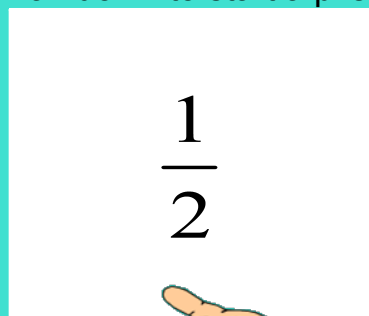
Try this...

1. Tear 1 piece of paper into 2 halves



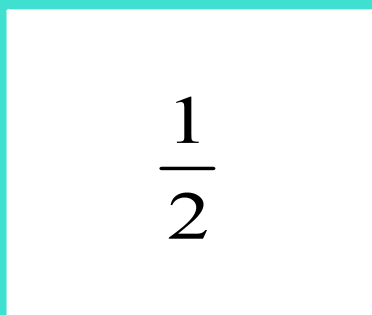
$$\frac{1}{2}$$

2. Label each piece & lay one half down to start a pile



3. Keep the other half in your hand

4. Tear the piece of paper in your hand into 2 halves



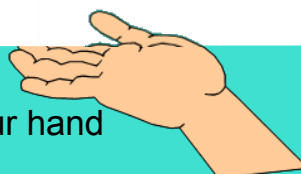
5. Label each piece & lay one half down in the pile

$$\frac{1}{2}$$


$$\frac{1}{?}$$



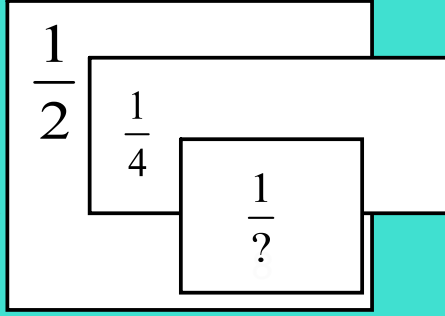
6. Keep one half in your hand



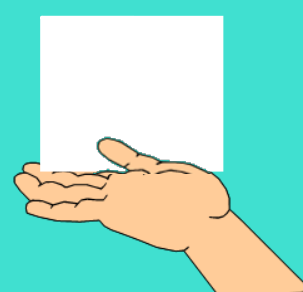
7. Tear the piece of paper in your hand into 2 halves



8. Label each piece & lay one half down in the pile



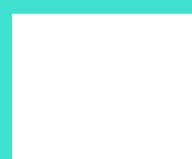
9. Keep one half in your hand



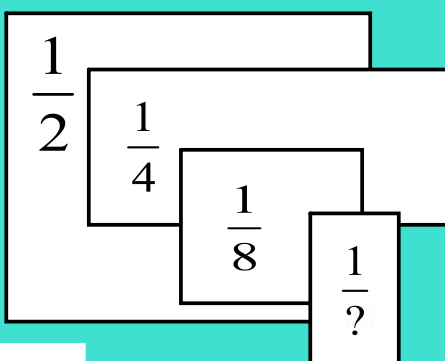
STOP Answer these questions:

- What is happening to the piece in your hand?
- What is happening in the pile?

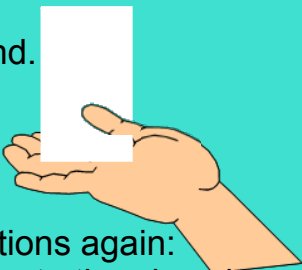
10. Tear the piece of paper in your hand into 2 halves.



11. Label each piece & lay one half down in the pile.



12. Keep one half in your hand.



STOP Answer these questions again:

- What is happening to the piece in your hand?
- What is happening in the pile?
- Will you ever be able to get rid of all of the paper in your hand?

Do you know the answer yet?

$$\frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \frac{1}{16} + \dots = ?$$

If not, continue to tear the paper you have in your hand, label one piece and place it in the pile, keep the other piece in your hand, and repeat.

When you know the answer, write it down on a scrap of paper and turn it over. Wait for further directions.

Try another one...

See if you can use the same technique to solve this problem. You'll need another sheet of paper.

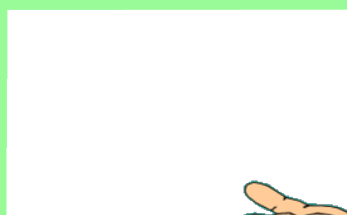
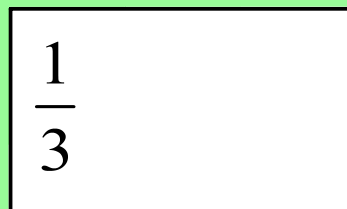
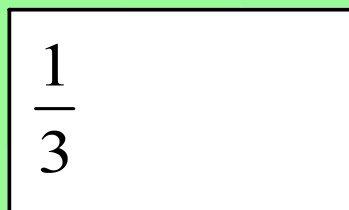
$$\frac{1}{3} + \frac{1}{9} + \frac{1}{27} + \frac{1}{81} + \dots = ?$$

Start by doing this...

1. Tear 1 piece of paper into thirds.

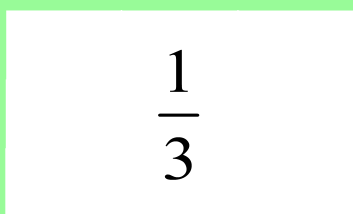


2. Label each piece & lay two thirds down to start 2 piles.

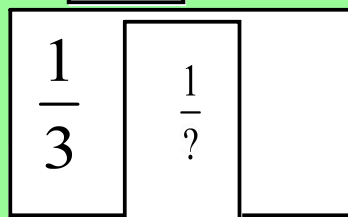
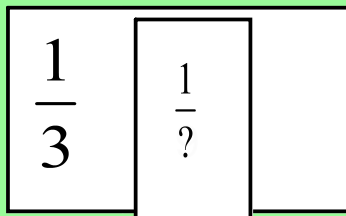


3. Keep the other third in your hand.

4. Tear the piece of paper in your hand into thirds.



5. Label each piece & add one piece in each pile.

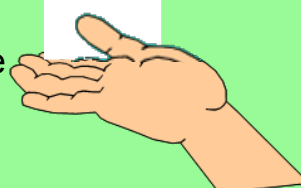


6. Keep the other piece in your hand.



Answer these questions:

- What is happening to the piece in your hand?
- What is happening in the pile?



7. Tear the piece of paper in your hand into thirds.

8. Label each piece & add one piece to each of the 2 piles.

9. Keep the other piece in your hand.

Ready to give the answer?
If not, keep tearing!

$$\sum_{n=1}^{\infty} \frac{1}{3^n} = \frac{1}{3} + \frac{1}{9} + \frac{1}{27} + \frac{1}{81} + \dots = ?$$

Can you answer these without paper?

Probably so, and very quickly based on what you've just learned. Try these. If you aren't sure, use a piece of paper, tear it, and check your answers!

$$\frac{1}{5} + \frac{1}{25} + \frac{1}{125} + \frac{1}{625} + \dots = ?$$

$$\frac{1}{4} + \frac{1}{16} + \frac{1}{64} + \frac{1}{256} + \dots = ?$$

$$\frac{1}{6} + \frac{1}{36} + \frac{1}{216} + \frac{1}{1296} + \dots = ?$$

$$\frac{1}{10} + \frac{1}{100} + \frac{1}{1000} + \frac{1}{10000} + \dots = ?$$

Got this? Impress a classmate or a parent!

Answers

If your students are ready, introduce summation notation when going over the answers.

Problem	Problem in Summation Notation	Limit or Series Answers
$\frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \frac{1}{16} + \dots = ?$	$\sum_{n=1}^{\infty} \frac{1}{2^n} = \frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \frac{1}{16} + \dots = ?$	1
$\frac{1}{3} + \frac{1}{9} + \frac{1}{27} + \frac{1}{81} + \dots = ?$	$\sum_{n=1}^{\infty} \frac{1}{3^n} = \frac{1}{3} + \frac{1}{9} + \frac{1}{27} + \frac{1}{81} + \dots = ?$	$\frac{1}{2}$
$\frac{1}{4} + \frac{1}{16} + \frac{1}{64} + \frac{1}{256} + \dots = ?$	$\sum_{n=1}^{\infty} \frac{1}{4^n} = \frac{1}{4} + \frac{1}{16} + \frac{1}{64} + \frac{1}{256} + \dots = ?$	$\frac{1}{3}$
$\frac{1}{5} + \frac{1}{25} + \frac{1}{125} + \frac{1}{625} + \dots = ?$	$\sum_{n=1}^{\infty} \frac{1}{5^n} = \frac{1}{5} + \frac{1}{25} + \frac{1}{125} + \frac{1}{625} + \dots = ?$	$\frac{1}{4}$
$\frac{1}{6} + \frac{1}{36} + \frac{1}{216} + \frac{1}{1296} + \dots = ?$	$\sum_{n=1}^{\infty} \frac{1}{6^n} = \frac{1}{6} + \frac{1}{36} + \frac{1}{216} + \frac{1}{1296} + \dots = ?$	$\frac{1}{5}$
$\frac{1}{10} + \frac{1}{100} + \frac{1}{1000} + \frac{1}{10000} + \dots = ?$	$\sum_{n=1}^{\infty} \frac{1}{10^n} = \frac{1}{10} + \frac{1}{100} + \frac{1}{1000} + \frac{1}{10000} + \dots = ?$	$\frac{1}{99}$

Teacher Notes

The object(s) of this lesson:

- explore an infinite problem
- introduce and discover the idea of a Limit
- explore geometric series,
- introduce/use summation notation, and/or
- learn how to iterate as an introduction to fractals.

The problem 1 page (pictured below) sets up the problem. Students will probably mention the idea of finding common denominators to try and solve this problem and it might be worth taking time to explore this. Click on the "Really" flipper button to reveal something on the back side.

Problem 1 Page

What's the answer?
Take a minute and think about how you might solve this problem!

$\frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \frac{1}{16} + \dots = ?$

You can find the answer if you have a piece of paper.

You'll need one piece of paper and a writing utensil. When you are ready, go to the next page and begin.

Really?

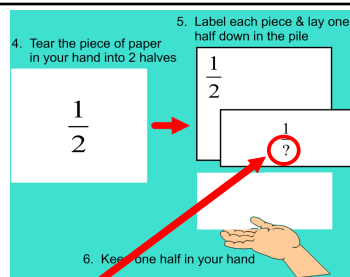
A note about the math:

A **sequence** is an ordered list of numbers. The **sum** of the terms of a sequence is called a **series**.

If a sequence of values follows a pattern of **multiplying a fixed amount** (not zero) times each term to arrive at the following term, it is referred to as a **geometric sequence**. The number multiplied each time is constant (always the same).

A **sequence** is an ordered list of numbers. The **sum** of the terms of a sequence is called a **series**. The fixed amount multiplied is called the **common ratio, r**, referring to the fact that the ratio (fraction) of the second term to the first term yields this common multiple. To find the common ratio, divide the second term by the first term.

<http://www.regentsprep.org/Regents/math/algtrig/ATP2/GeoSeq.htm>



When students have labeled their pieces with fractions, have one of them click on the question marks to reveal the correct fraction. This works on each page where there is a question mark in the fraction.

As students build piles with their pieces of paper (fractions), each pile will represent the problem they are trying to solve, while the paper in their hand keeps getting smaller (approaching zero).

Also, notice that the piles are the equal both in amounts of paper and fractions that are represented. The original 1 piece of paper is being sorted into these piles. So, if there are two piles, each represents 1/2 of the whole piece of paper. If there are three piles, each pile is the problem being solved and 1/3 of the original piece of paper.

In mathematics, a **limit** is the value that a function or **sequence** "approaches" as the input or index approaches some value. - James Stewart (2008), *Calculus: Early Transcendentals* (6th ed.).

Iteration means the act of repeating a process with the aim of approaching a desired goal, target or result. Each repetition of the process is also called an "iteration," and the results of one iteration are used as the starting point for the next iteration.

Start the Investigation!

Check your answers

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